S E S 2 0 1 9
Fifteenth International Scientific Conference
SPACE, ECOLOGY, SAFETY
6 – 8 November 2019, Sofia, Bulgaria

MODELING MAGNETIC FIELDS OF ACCRETION DISCS IN BINARY STARS USING NO-Z APPROXIMATION

Daniela Boneva[1], Evgeny Mikhailov[2], Maria Pashentseva[2]

[1]Space Research and Technology Institute – Bulgarian Academy of Sciences
[2]M. V. Lomonosov Moscow State University
e-mail: danvasan@space.bas.bg; ea.mikhajlov@physics.msu.ru; marusya_pashenceva@mail.ru

Keywords: *accretion discs; magnetic fields; binary stars;*

Abstract: *It is well-known that the magnetic fields can be important for studying the processes in the accretion discs. The disc evolution is connected with differential rotation, turbulence and convection of highly conductive medium in the disc. It can be quite convenient to study the processes of the magnetic field grows using no-z approximation, which is well known for the galaxy dynamo. We try to show the field evolution for accretion discs around white dwarfs in binary stars using both Keplerian law and some modified rotation curves. It is shown, that the magnetic fields can grow for quite small values dimensionless dynamo governing parameters (if we compare with galaxies). We give typical plots of the solutions of different cases.*

NO-Z ПРИБЛИЖЕНЕ ЗА МОДЕЛИРАНЕ НА МАГНИТНОТО ПОЛЕ В АКРЕЦИОНЕН ДИСК НА ДВОЙНА ЗВЕЗДНА СИСТЕМА

Даниела Бонева[1], Евгений Михайлов[2], Мария Пашенцева[2]

[1]Институт за космически изследвания и технологии – Българска академия на науките
[2]Московски Държавен Университет, М.В. Ломоносов
e-mail: danvasan@space.bas.bg; ea.mikhajlov@physics.msu.ru; marusya_pashenceva@mail.ru

Ключови думи: *акреционни дискове; магнитни полета; двойни звезди;*

Резюме: *Известно е, че магнитните полета могат да бъдат важни за изучаването на процесите в акреционните дискове. Еволюцията на диска е свързана с диференциалното въртене, турбулентността и конвекцията на силно проводяща среда в диска. Процесите на разрастващото се магнитно поле биха могли да се изследват като се използва приближение по z, което е добре известно при галактиктичното динамо. Ние представяме еволюцията на полето на аккреционни дискове около бели джуджета в двойни звезди, като използваме както закона на Кеплер, така и някои модифицирани криви на въртене. Показано е, че магнитните полета могат да нарастват при доста малки стойности на безразмерните параметри, управляващи динамото (ако сравним с галактиките). Представени са типични графики с решенията за различните случаи.*

Introduction

During the binary stars' evolution, accretion discs are appeared to be one of the most powerful phenomena among the astrophysical objects (Frank et al., 2002; Spruit, 2010; Pringle, 1981). A basic role in the disc's physical processes and evolution take magnetic fields (Shakura & Sunyaev, 1973). Partially, an angular momentum transfer, which is the most important mechanism of the accretion disc existence, could be explained by the magnetic field activity. Its generation is usually described by the dynamo mechanism, based on both processes the alpha – effect and differential rotation (Ruzmaikin et al., 1988, Stepinski & Levy, 1990; Zeldovich et al., 1983). The generation of magnetic field and its operation in accretion discs has been the subject of study for years (Tout & Pringle, 1996, Zhilkin & Bisikalo, 2010).

The magnetic fields are thought to play an important role in evolution of accretion discs (Shakura & Sunyaev 1973). For example, they should describe the transition of the angular momentum and another effects. We can assume that generation of this field is connected with dynamo mechanism, as in other astrophysical objects, such as Sun, stars, galaxies etc. (Sokoloff et al., 2014). Dynamo describes the transition of the energy of the turbulent motions to the energy of the magnetic field. It is based on joint action of two different processes. The first one is the alpha-effect which describes vorticity of the turbulent motions, and the second is the differential rotation (most of the astrophysical objects rotate not as a solid state).

This mechanism can be studied by direct numerical simulation, but it is connected with different problems. From the one hand, it is connected with very large computational resources. From the other hand, the magnetic field will depend on different kinematic parameters not very transparently. We shall have a set of the results, which are quite difficult to be analysed. So, it would be useful to have a simple model of field, which gives us an opportunity to show the field evolution, using several well-known characteristics of the object.

As for galaxies, where no-z approximation is widely used, it has been developed for thin discs (Moss 1995; Subramanian & Mestel, 1993). For such objects, the magnetic field approximately lies in the equatorial plane. The z-derivatives of the field can be changed by the algebraic expressions, or taken from the solenoidality condition for the magnetic field. One of the main advantages of this model is that we can describe only two components of the field which depend on only two different coordinates (as for the axi-symmetric case we can reduce this dependence to one coordinate).

It would be quite effective to use the no-z approximation for modeling the magnetic fields of galaxies (Moss et al., 2016). Firstly, it is necessary to describe the dynamo governing parameters for such objects. Secondly, we should take into account the flows of the medium in the accretion discs. Also, it is necessary to revise all main characteristics of the model to make it applicable for accretion discs.

The main aim of our work is to describe the magnetic field in accretion discs using no-z approximation. Also we take the nonlinear modification of the equations, and typical differences from the linear case. In the next sections we present the basic equations and the results of numerical calculations.

Basic equations and parameters of the accretion disc

In the case of a thin disc approximation, we assume that the matter moves with angular velocity Ω. If we accept that the velocity has a Keplerian value (Frank et al. 2002) (it's coming from the requirement that Keplerian velocity should be highly supersonic - $c_s \ll (GM/R)^{1/2}$), then: $\Omega = \Omega_K(R) = \left(\frac{GM_1}{R_*}\right)^{1/2}$, where

M_1- the mass of the accreting star (depends on the studied object); R_*- radius of the compact object (for the white dwarf primary, the average value of the radius is usually accepted to be $\leq 10^9$ cm.)

In the case of steady thin disc approximation, again, for the radial velocity we can write (Frank et al. 2002):

$$v_r = -\frac{3\nu}{2r}\left[1 - \left(\frac{r_{min}}{r}\right)^{\frac{1}{2}}\right]^{-1},$$

where r_{min} expresses the inner disc radius R_{ind}, r - the distance to the object, $\nu = \frac{lv}{3}$ - the viscosity.

The values of the inner radius of the accretion disc vary, depending on the velocity and radius of the white dwarf star (as an example: inner disc radius $R_{ind} \approx 3\ R_{wd} - 5R_{wd}$). Adopting that the radius of the white dwarf is $R_{wd} \approx 0.0118 R_{\odot}$ (solar radius), which is the radius of the primary star in dwarf nova V2051 Oph (Ophiuchi), and accepting the Keplerian character of the disc, we have $\sim 3.87 R_{wd}$ (Baptista et al.,1998).

When we study the magnetic fields in accretion disc around white dwarfs in binary systems, we will use the approach which is quite similar with the galaxy dynamo.

According to the dynamo theory, the magnetic field's evolution is characterized by the Steenbeck - Krause –Radler equations (Steenbeck et al. 1966) or the mean-field dynamo equations.

(1) $$\frac{\partial \vec{B}}{\partial t} = rot(\alpha \vec{B}) + rot[\vec{V}, \vec{B}] + \nu \Delta \vec{B},$$

where $\vec{B}$ is the magnetic field, α is the alpha-parameter - describes the alpha-effect, $\vec{V}$ is the large-scale velocity (usually connected with rotation), and ν is the turbulent diffusivity. This vector equation contains three scalar equations, which are quite difficult to be solved. Here we describe the simplification of these equations using the approaches of no-z approximation for accretion discs.

For alpha-effect we can take the model (Moss, 1995):

(2) $\alpha = \alpha_0(r)\frac{z}{h}$;

where h is the half-thickness of the disc, and $\alpha_0(r)$ is the typical value of the alpha-effect for r. The alpha-effect is connected with the Coriolis force and the coefficient is proportional to the angular velocity of the rotation (Arshakian et al., 2009): $\alpha \propto \Omega$.

We assume that the z-component of the magnetic field is quite small, and for another parts we can take:

(3) $B_r(r,z,t) = B_r(r,0,t)\cos\left(\frac{\pi z}{2h}\right)$;

$$B_\varphi(r,z,t) = B_\varphi(r,0,t)\cos\left(\frac{\pi z}{2h}\right).$$

The system of equations for the magnetic field component in the disc plane is obtained to be:

(4) $$\frac{\partial B_r}{\partial t} = -\frac{\alpha}{h}B_\phi + \eta\left(\frac{\partial^2 B_r}{\partial z^2} + \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rB_r)\right)\right)$$

$$\frac{\partial B_\phi}{\partial t} = r\frac{d\Omega}{dr}B_r + \eta\left(\frac{\partial^2 B_\phi}{\partial z^2} + \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rB_\phi)\right)\right),$$

where η is the magnetic diffusivity, h - the disc half-thickness, r - the distance to the disc centrum, Ω is the angular velocity.

The second derivatives of the magnetic field on the perpendicular to the disc direction can be replaced by algebraic expressions as follows (Moss & Sokoloff, 2013):

(5) $$\frac{\partial^2 B_r}{\partial z^2} = -\frac{\pi^2 B_r}{4h^2}$$

$$\frac{\partial^2 B_\phi}{\partial z^2} = -\frac{\pi^2 B_\phi}{4h^2}$$

So, the equations of the field will be the following (Moss & Sokoloff 2013):

(6) $$\frac{\partial B_r}{\partial t} = -\frac{\alpha_0}{h}B_\phi - \eta\frac{\pi^2 B_r}{4h^2} - V_r\frac{\partial B_r}{\partial r} + \eta\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rB_r)\right);$$

$$\frac{\partial B_\varphi}{\partial t} = r\frac{d\Omega}{dr}B_r - \eta\frac{\pi^2 B_\varphi}{4h^2} - V_r\frac{\partial B_\varphi}{\partial r} + \eta\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rB_\phi)\right),$$

where V_r is the radial velocity of the flow of the medium.

After some reconstruction and including the dimensionless parameters, as the α effect, differential rotation and magnetic field dissipation in the disc plane: $R_\alpha = \frac{h\alpha(R)}{\eta}$, $R_\omega = \frac{\Omega_0(R)h^2}{\eta}$ and $\lambda = \frac{h}{R}$, the non-z approximation system of equations takes the form:

(7) $$\frac{\partial B_r}{\partial t} = -\frac{R_\alpha}{r^{3/2}}B_\phi - \frac{\pi^2 B_r}{4} - V_r\frac{\partial B_r}{\partial r} + \lambda^2\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rB_r)\right);$$

$$\frac{\partial B_\phi}{\partial t} = -\frac{3R_\omega}{2r^{3/2}}B_r - \frac{\pi^2 B_\phi}{4} - V_r\frac{\partial B_\varphi}{\partial r} + \lambda^2\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rB_\phi)\right).$$

Here we measure time in units of $\frac{h^2}{\eta}$, the distances are measured in R.

The magnetic field growth cannot grow infinitely. Its value is restricted by the equipartition value $B^*(r)$. If we measure the magnetic field in units of $B^*(R)$, the equations will be:

(8) $$\frac{\partial B_r}{\partial t} = -\frac{R_\alpha}{r^{3/2}}\left(1 - \frac{B_r^2 + B_\varphi^2}{B^{*2}(r)}\right)B_\phi - \frac{\pi^2 B_r}{4} - V_r\frac{\partial B_r}{\partial r} + \lambda^2\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rB_r)\right);$$

$$\frac{\partial B_\phi}{\partial t} = -\frac{3R_\omega}{2r^{3/2}} B_r - \frac{\pi^2 B_\phi}{4} - V_r \frac{\partial B_\varphi}{\partial r} + \lambda^2 \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(rB_\phi\right)\right).$$

We take the value of the disc inner radius R_{ind} to be large enough. That could be further useful for studying a wide range of phenomena.

Results

The problem in this paper was solved numerically using finite – difference method with zero outer disc's boundary conditions: $B_{r=r_{min}} = B_{r=r_{max}} = 0$

The behavior of the magnetic field is characterized by a dynamo number $= R_\alpha R_\omega$, which describes both the possibility of field growth and the rate of this growth (if present). Note that the magnetic field increases with the increase of D. This way the dynamo effect is threshold. The magnetic field can grow exponentially when the dynamo exceeds a certain critical value Dcr determined by the properties of the model. Thus, it increases at $D > Dcr$ and decays at $D < Dcr$. In similar models for galaxies, it was found that $Dcr = 7$. In the case of an accretion disk, a numerical calculation shows that $Dcr = 0.112$, i.e. the critical dynamo is noticeably lower. It can be shown that $D \sim h^2 \Omega^2 v^{-2}$ (Arshakian et al., 2009).

The evolution of the field over time is presented in Figure 1.

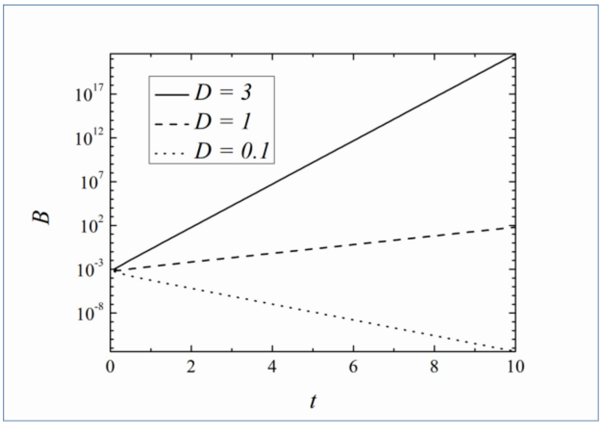

Fig. 1. The linear case of the magnetic field $\boldsymbol{B}$ time dependence for 3 different values of the dynamo number $\boldsymbol{D}$ (D = 0.1, 1, 3)

In the frame of linear models, the results show that the magnetic field can increase unlimited (as in Fig. 1), which is contrary to the physics sense. This requires the use of a model in which the saturation is counted. In this case, for similar values of the parameters and taking into account the nonlinearity of the alpha - effect, the magnetic field does not increase unlimitedly. It reaches the level of equal distribution, after which the increase stops. The saturation can be estimated by introducing a nonlinear modification of the coefficient responsible for the alpha effect:

$$R_\alpha \to R_\alpha \left(1 - \frac{\left(B_r^2 + B_\varphi^2\right)}{B^{*2}}\right)$$

Then the radial structure of magnetic field with nonlinearity is looks like as in the Fig. 2 below. Fig. 2 shows the case of nonlinear equations. Here we can see the characteristic dependence of magnetic field for different inner radius. For smaller inner radius the maximum field will be larger.

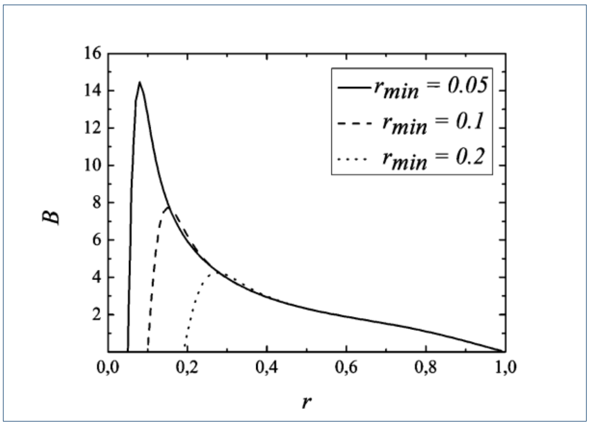

Fig. 2. Magnetic field $\boldsymbol{B}$ against the distance to disc centrum **r**, for 3 different inner radius of the disc r_{min}. The non-linearity of the α - effect is shown in the calculations.

The values and results are obtained based on the data parameters of Dwarf Nova binary star IP Pegasi (Marsh, 1988; Steeghs et al., 1997).

Conclusion

In this paper, we have modelled the magnetic field of the accretion discs using no-z approximation, which was historically developed for galaxies.

We have obtained the time dependence of the field, its radial structure and the critical dynamo number.

It was shown that the field can grow much faster than in the galaxies (in dimensionless units of time). From our point of view, it is connected with another rotation law, which describes larger gradients.

References:

1. Arshakian, T. G., R. Beck, M. Krause, D. Sokoloff, 2009, AA, 494, 21.
2. Baptista, R., M. S. Catalan, Keith Horne, D. Zilli, 1998, MNRAS, V.300, 1, pp. 233.
3. Frank, J., A. King, D. Raine, 2002, Accretion Power in Astrophysics, 3-rd ed., Cambridge Univ. Press, New York
4. Marsh, T. R., 1988, MNRAS, 231,1117-1138.
5. Moss, D., 1995, MNRAS, V.275, p. 191–194.
6. Moss, D., Sokoloff D., 2013, GAFD, 107, 494.
7. Moss, D., E. Mikhailov, O. Silchenko et al., 2016a, A&A, V. 592, A44.
8. Pringle, J. E., 1981, Accretion disks., ARA&A,19: 137–162.
9. Ruzmaikin, A. A., D. D. Sokolov, A.M. Shukurov, 1988, Magnetic Fields of Galaxies (Nauka, Moscow, Kluwer, Dordrecht,), p. 280.
10. Shakura, N. I., R. A. Sunyaev, 1973, A & A, V. 24. pp. 337.
11. Sokoloff, D. D., R. A. Stepanov, P.G. Frick, 2014, Phys. Usp., 57, 292.
12. Spruit, H., 2010, “Accretion Disks” in The Encyclopedia of Astronomy and Astrophysics, IOP Publishing Ltd
13. Steeghs, D., E. T. Harlaftis, Keith Horne, 1997, MNRAS, V. 290, 2, L28.
14. Stepinski, T. F., E. H., Levy, 1990, ApJ, 362: 318–332
15. Steenbeck, M., F. Krause, K.H. Radler, 1966, Z. Naturforsch. 21a, 369–376. (or 1971, The turbulent dynamo, Tech. Note 60, NCAR, Boulder, Colorado).
16. Subramanian, K., L. Mestel., 1993, MNRAS. V.265. pp. 649–654.
17. Tout, A. Chr.,J. E.Pringle, 1996, *MNRAS*, V. 281, Issue 1, pp. 219–225.
18. Zhilkin, A. G., D. V. Bisikalo, 2010, Astron. Rep., 54, 840.
19. Zeldovich, Ya. B., A. A. Ruzmaikin, D. D. Sokolov, 1983, Magnetic Fields in Astrophysics. Gordon and Breach Science Publishers.